Age-graded results.

By

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In the evaluation of any performance it is usual to compare a recorded time or distance against a standard. In the Open category, the natural standard is the world record at the relevant distance. An athlete's lifetime best is typically set during an interval of ten to twenty years. Before and after this period, any reference to the Open standard is not entirely valid, because it is not a fair comparison of like with like.

This issue provides the motivation for the development of a different set of standards, a set based on the athlete's age. The past decade has witnessed increasing interest in the appropriate methodology for the construction of age-based standards which would lead to the calculation of age-graded times.

This levelling of the age-graded playing- field permits a direct comparison with the Open times. The ratio of Open to age-graded times produces a percentage, the age-graded percentage or score, a dimensionless number, which allows a comparison, not only across age groups, but also across events of different lengths, as well as between genders.

Interpolation.

Age-grading factors are points measured over a 2-dimensional response surface generated by a distance dimension and an age dimension. The gradient of the surface is relatively steep in the age dimension but fairly flat over distances. The measurement interval between races is variable, this interval being a function of the standard distances. The most popular recognised distances are represented explicitly. These are, for the distance events: 1 mile, 2 km, 3 km, 2 miles, 4 km, 3 miles, 5 km, 6 km, 4 miles, 8 km, 5 miles, 10 km, 12 km, 15 km, 10 miles, 20 km, 21.1 km, 25 km, 30 km, 42.2 km, 50 km, 50 miles, 100 km, 100 miles, 150 km and 200 km. Male and female event lists are identical and no distance interpolation is needed for these distances.

Factors for non-standard distances, such as the Dartmouth Natal Day 6 miler, or arbitrary distances, characteristic of stage races, such as the Cabot Trail Relay, require interpolation along the distance dimension. In the case of the 6 miler, interpolation is carried out between 5 miles and 10 km. The corresponding Open standard, or world record or best, where none is recognised or exists, is also obtained in an analogous fashion.

In practice interpolation is almost always required along the age dimension. The sole exception is the case where the athlete's age exactly coincides with an age level in the program. For the 1994 factors, 5 year intervals are used for the most part, whereas 1-year intervals are employed throughout the 2006 factors.

Where both distances and ages must be interpolated, a 2-dimensional interpolation formula is applied over the factor response surface.

MATHEMATICAL METHOD

Let the event distances be defined as:

d(i) = distance in km for event i (explicit)
d(i+1) = distance in km for event i+1 (explicit)
d(k) = distance in km for event k (arbitrary)
$d(i) < d(k) < d(i+1)$

and the athlete ages be given as:

a(j) = athlete age in years and decimal of years for age level j
a(j+1) = athlete age in years and decimal of years for age level j+1
a(l) = ditto arbitrary age l
$a(j) < a(l) < a(j+1)$

and the age-graded factor response surface at distance d(k) and athlete age a(l) as:

$$y(k,l) = f[\, d(k), a(l)\,]$$

Then the distance interpolation factor, $\lambda(\kappa)$, for distance d(k), expressed as a fraction of the interval between the bounding distances is given by:

$$\lambda(\kappa) = \begin{cases} [\, d(k) - d(i)\,] / [\, d(i+1) - d(i)\,], & d(i) < d(k) < d(i+1) \\ 0, & d(i) = d(k) \end{cases} \quad (1)$$

with the property: $0 < \lambda(\kappa) < 1$,

and the athlete age interpolation factor, $\mu(l)$, for age a(l), expressed as a fraction of the interval between the bounding ages is given by:

$$\mu(l) = \begin{cases} [\, a(l) - a(j)\,] / [\, a(j+1) - a(j)\,], & a(j) < a(l) < a(j+1) \\ 0, & a(j) = a(l) \end{cases} \quad (2)$$

with the property: $0 < \mu(l) < 1$.

These fractional weights may now be combined in a 2-dimensional convex combination to yield the interpolated point, y(k,l), on the age-graded response surface:

$$
\begin{aligned}
y(k,l) = f[\, d(k), a(l)\,] &= (1 - \lambda(\kappa))\ \{ \mu(l)\, f[\, d(i,a(j+1)\,] - (1 - \mu(l))\, f[\, d(i), a(j)\,] \} \qquad (3)\\
&\quad + \lambda(\kappa)\ \{ \mu(l)\, f[\, d(i+1),a(j+1)\,] - (1 - \mu(l))\, f[\, d(i+1), a(j)\,] \}\\
&\qquad\qquad d(i) < d(k) < d(i+1),\ \ a(j) < a(l) < a(j+1)\\
&= y(i,j) = f[\, d(i), a(j)\,],\ \ d(k) = d(j),\ a(l) = a(j)\\
&= 1, \qquad\qquad \text{otherwise.}
\end{aligned}
$$

This is the age-graded factor obtained by interpolation.

The interpolated Open standard, r(k), is obtained by:

$$r(k) = (1 - \lambda(\kappa))\ r(i) + \lambda(\kappa)\ r(i+1), \quad r(i) < r(k) < r(i+1) \qquad (4)$$

where r(i), r(i+1) are the Open standards for events of lengths d(i) and d(i+1) respectively.

The age-graded standard, R(k,l), is the quotient of the Open standard and the age-graded factor:

$$R(k,l) = r(k) / y(k,l). \qquad (5)$$

Age-graded time, T(k,l), is the product of t(k), observed elapsed time, gun or chip, for event k, measured in seconds, and y(k,l), the age-graded factor for distance d(k) and athlete age a(l):

$$T(k,l) = t(k)\ y(k,l). \qquad (6)$$

Then the age-graded percentage, S(k,l), is the ratio of the Open standard and the age-graded time, expressed as a percentage:

$$S(k,l) = 100\ [\ r(k) / T(k,l)\ ]. \qquad (7)$$

S(k,l) may exceed 100 where a new performance produces a time which is substantially lower than the previous age category times, which were used in the calculation of the factors on the age-graded response surface.

WORKED EXAMPLES USING THE 2006 FACTORS.

1. Dartmouth 6 Miler, 7 August, 2006, Denise Robson, 34:21.

This is a fairly straightforward case to start with, because Denise is too young to have much of an age-graded factor, it being close to 1.0 for all events. For age 37 it is 0.9816 and for 38 0.9768, for both bounding distances. Denise was 37.715 when she ran. Since 6 miles is a non-standard distance, however, interpolation is also required for distance and the Open standard.

Distance interpolation occurs between 8.045 and 10km, as 6 miles is 9.654km. Hence, equation (1) is evaluated as the proportion

$$\lambda(\kappa) = (9.654 - 8.045) / (10.0 - 8.045) = 0.82302$$

the denominator being the entire length of the interpolation interval, and the numerator shows the segment represented by the 6 miles.

Equation (2) is evaluated as the age proportion

$$\mu(l) = (37.715 - 37.0) / (38.0 - 37.0) = 0.715$$

and equation (4) yields

$$\begin{aligned} y(k,l) &= (1.0 - 0.82302) \{ (0.715)\, 0.9768 + (1.0 - 0.715)\, 0.9816 \} \\ &\quad + (0.82302) \{ (0.715)\, 0.9768 + (1.0 - 0.715)\, 0.9816 \} \\ &= (0.17698) \{ 0.978168 \} + (0.82302) \{ 0.978168 \} \\ &= 0.978168 \end{aligned}$$

The Open standard for 5 miles, female, is r(i) = 1452 seconds, or 24:12, that for 10km is r(i+1) = 1820, or 30:20. Equation (4) is evaluated as

$$r(k) = (1.0 - 0.82302)\, 1452 + (0.82302)\, 1820 = 1754.871 \text{ seconds, or } 29{:}15.$$

This is the imputed female Open standard for 6 miles obtained by interpolation.

Denise ran 34:21 or 2061 seconds.

Since y(k,l) = 0.978168 for Denise, which is close to 1.0, equations (5), and (6) change slightly. The age-graded standard, R(k,l), equation (5)

$$R(k,l) = r(k) / y(k,l) = 1754.871 / 0.978168 = 1794.03 \text{ seconds, or } 29{:}54$$

Her age-graded time, T(k,l), equation (6), is calculated as

$$T(k,l) = 2061\, (0.978168) = 2016.004 \text{ seconds or } 33{:}36.$$

Then the age-graded percentage, S(k,l), equation (7), results in

S(k,l) = 100.0 (1794.03 / 2061) = 87.05%.

2. Richard Beazley 6 Miler, Hantsport, 1 July, 2005, Michael Wills, 37:58.

Staying with the 6 miler for the moment, so that the distance interpolation remains the same, it is instructive to see the effect of adding a significant age grading factor. Again the age-graded factors for 5 miles and 10km are the same. For age 59 it is 0.8113 and for 60 0.8043. Michael was 59.95 when he ran.

Equation (2) is evaluated as the age proportion

μ(l) = (59.95 - 59.0) / (60.0 - 59.0) = 0.95

and equation (4) comes out as

y(k,l) = (1.0 - 0.82302) { (0.95) 0.8043 + (1.0 - 0.95) 0.8113 }
+ (0.82302) { (0.95) 0.8043 + (1.0 - 0.95) 0.8113 }
= (0.17698) { 0.80465 } + (0.82302) { 0.80465 }
= 0.80465

This is the interpolated age group factor for 6 miles.

The Open standard for 5 miles, male, is r(i) = 1280 seconds, or 21:20, that for 10km is r(i+1) = 1611, or 26:51. Equation (4) is evaluated as

r(k) = (1.0 - 0.82302) 1280 + (0.82302) 1611 = 1552.41962 seconds, or 25:52.

This is the imputed male Open standard for 6 miles obtained by interpolation.

The age-graded standard, R(k,l), equation (5)

R(k,l) = r(k) / y(k,l) = 1552.41962 / 0.80465 = 1929.3104 seconds, or 32:09.

Michael ran 37:58 or 2278 seconds.

His age-graded time, T(k,l), equation (6), is calculated as

T(k,l) = 2278 (0.80465) = 1832.9927 seconds or 30:33.

Then the age-graded percentage, S(k,l), equation (7) is

S(k,l) = 100 [1552.41962 / 1832.9927] = 84.69%.

3. Cabot Trail Relay, 27 May, 2006, Leg 5 (17.5km), Ed Whitlock, 1:14:35.

This is a completely arbitrary distance, which can, however, be interpolated between standards for 10 miles and 20km. Furthermore, unlike the 6 miler case, the age-graded factors are different for the two distances, and Ed (birthday 19310306) is between exact ages in years, so this is the most general case. The age-graded factor for age 75 in the 10 miles is 0.684 and in the 20km is 0.6858; for 76 the corresponding factors are 0.6722 and 0.6742. Ed was 75.2245 when he ran.

Hence, equation (1) is evaluated as the proportion

$\lambda(\kappa)$ = (17.5 - 16.09) / (20.0 - 16.09) = 0.3606

Equation (2) is evaluated as the age proportion

μ(l) = (75.2245 - 75.0) / (76.0 - 75.0) = 0.2245

and equation (4) comes out as

y(k,l) = (1.0 - 0.3606) { (0.2245) 0.6722 + (1.0 - 0.2245) 0.684 }
+ (0.3606) { (0.2245) 0.6742+ (1.0 - 0.2245) 0.6858}

= (0.6394) { 0.6813509 } + (0.3606) { 0.6831958 }
= 0.682016

This is the interpolated age group factor for 17.5km.

The Open standard for 10 miles, male, is r(i) = 2663 seconds, or 44:23, that for 20km is r(i+1) = 3358, or 55:58. Equation (4) is evaluated as

r(k) = (1.0 - 0.3606) 2663 + (0.3606) 3358 = 2913.617 seconds, or 48:33.

This is the imputed male Open standard for 17.5km obtained by interpolation.

The age-graded standard, R(k,l), equation (5)

R(k,l) = r(k) / y(k,l) = 2913.617 / 0.682016 = 4272.065 seconds, or 1:11:12.

Ed ran 1:14:35 or 4475 seconds.

So his age-graded time, T(k,l), equation (6), is calculated as

T(k,l) = 4475 (0.682016) = 3052.2185 seconds or 50:52.

Then the age-graded percentage, S(k,l), equation (7), is

$S(k,l) = 100 [2913.617 / 3052.2185] = 95.45\%$.