Estimated Age Effects in Athletic Events and Chess

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Abstract

Rates of decline are estimated using record bests by age for chess and for various track and field, road running, and swimming events. Using a fairly flexible functional form, the estimates show linear percent decline between age 35 and about age 70 and then quadratic decline after that. Chess shows much less decline than the physical activities. Rates of decline are generally larger for the longer distances, and for swimming they are larger for women than for men. An advantage of using best-performance records to estimate rates of decline is that the records are generally based on very large samples. In addition, the age range is large. In this study the age range is 35–100 for swimming, 35–98 for track and field and running, and 35–94 for chess. The estimates also do not suffer from traditional forms of selection bias.

Over 80 years ago Hill (1925) pointed out the potential usefulness of athletic records to study the physiology of muscular exercise. He noted that athletic events are really experiments on subjects under tightly controlled conditions and that the results are a “collection of natural constants of muscular effort in the human race”

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(p. 481). Moore (1975) was the first to use best-performance records by age to examine how athletic performance changes with age. This was followed by Salthouse (1976). Stones & Kozma (1980) used records by five-year age intervals to examine performance changes by age—see also Stones & Kozma (1981, 1982, 1986a, 1986b). The next study after Salthouse (1976) to use records by yearly age intervals was Fair (1994). More recent studies using athletic records include Baker, Tang, & Turner (2003), Donato, Tench, Glueck, Seals, Eskurza, & Tanaka (2003), Tanaka & Seals (1997), and Tanaka & Seals (2003). An advantage of using athletic records to examine human performance (aside from the controlled conditions stressed by Hill) is that most of them are based on very large samples. For example, many 60-year old men have run a marathon, and so the fastest marathon time ever recorded by a 60-year old man is based on a very large sample of attempts, much larger than would ever be feasible in an experimental setting. In addition, the age range for which records exist is large, again much larger than is feasible in an experimental setting.

This study extends the results in Fair (1994). The athletic events have been extended to include swimming for both men and women, and one cognitive event has been added: chess. In addition, there are now better data at the old ages on track and field events and road running events because of the expanded participation in these events by the old. The age range used in this study is 35–100 for swimming, 35–98 for track and field and running, and 35–94 for chess.

Using age records to examine performance changes by age is likely to lessen selection bias problems. In psychology selection bias is a common problem in cross-section studies of cognitive aging because more talented people may be over represented at the old ages (Anstey, Hofer, & Luszcz, 2003, Brant & Fozard, 1990, Hertzog & Nesselroade, 2003, Lindenberger & Baltes, 1997). Selection bias is also common in cross-section studies of $\dot{V}o_{2max}$ aging effects in physiology (Dehn & Bruce, 1972, Pollock, Foster, Knapp, Rod, & Schmidt, 1987). Selection bias may also exist in longitudinal studies if weaker subjects drop out of the study more frequently than stronger ones (Colshen & Wallace, 1991, Tanaka & Seals, 1997). When best-ever performances by age are used, it does not matter that, say, the percent of talented people in the 70-year-old sample is larger than the percent in the 40-year-old sample because only the performance of the very best person by age is used. It also does not matter if, say, fewer 70-year-olds than 40-year-olds train hard and compete as long as some of the best at both ages compete. Again, in the end only the one best performance per age is used. It may be, of course, that the estimated performance declines using age records are not representative of declines for the average person, and in this sense the selection using age records may be misleading regarding the average person. This is discussed later.

METHODS

The method used in Fair (1994) uses a more flexible functional form than was used in previous studies and deals with two important statistical problems that were not considered. (As discussed below, the two statistical problems are accounting for dominated times and for the fact that all measurement errors are non negative.) The functional form postulates a linear *percent* rate of decline between age 35 and some transition age, which is estimated, and then quadratic decline after that. This functional form is more flexible because it allows the transition age to be estimated along with the other parameters. If, for example, quadratic decline begins soon after 35, then the estimated transition age will be close to 35 and there will be almost no range of linear decline. On the other hand, if quadratic decline begins late in life, the estimated transition age will be close to the end of the age range. The use of percentage rates of decline means that the rates are unit free.[1] The estimates in this paper would, of course, be affected if a different functional form were used.

The estimates are based on the following three assumptions: 1) decline has begun by at least age 35, 2) the rate of decline is the same per year between age 35 and some transition age k^* (i.e., linear rate of decline), and 3) the rate of decline increases by the same amount per year after the transition age (i.e., quadratic rate of decline). b_k will be used to denote the log of the biological minimum time for age k for the particular event.[2] The exact equation that is postulated for b_k, based on the above three assumptions, is presented in the appendix.

b_k, the log of the biological minimum time, is not necessarily observed for a given age and event. r_k will be used to denote the log of the *observed* record time for age k. By definition,

$$r_k = b_k + \epsilon_k \quad , \tag{1}$$

where ϵ_k denotes the measurement error. This error will be close to zero if the

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[1] Baker, Tang, & Turner (2003) is an example of a study using percentage rates of decline.

[2] For the high jump the measure of performance is distance and for chess the measure is rating, where, unlike for time, larger is better than smaller. For simplicity, the following is written assuming time is the measure, but the switch to distance or rating is straightforward. Again, because of the use of percentages (logarithms), it does not matter whether the measure is in units of time, distance, or rating.

record time is close to the biological minimum. If a large number of people of age k have competed in the event, the record time is likely to be fairly close to biological minimum and thus the measurement error close to zero. If, on the other hand, the number who have competed is fairly small, as it is at the very old ages, the record time may be above the biological minimum and thus the error positive. This problem of a positive measurement error will be called the "small sample problem."

One way in which the small sample problem may manifest itself is for the record time at some age to be larger than the record time at an older age. If this is true, the record time at the younger age will be called a "dominated time." This is the first statistical problem mentioned above: how to deal with dominated times? The procedure used in the estimation work is simply to exclude observations that are dominated. Under the assumption that people never get better after age 35, a dominated time cannot have a zero measurement error. Excluding these observations avoids using values that for sure have positive measurement errors. Note, however, that although dominated times are "soft" in that they are likely to be broken in the future, a non-dominated time may also be soft, especially at the very old ages. In other words, excluding dominated times does not necessarily eliminate all small sample problems.

The exact equation that was estimated is presented in the appendix, along with a discussion of the estimation method that was used. The estimation method is designed to insure that all the estimated errors are non negative. This deals with the second statistical problem mentioned above.

Estimates are presented in Table 1 for 1) the rate of decline up to the transition age, denoted α, 2) the transition age, denoted k^*, and 3) the quadratic parameter, denoted δ. The quadratic parameter is the amount by which the rate of decline changes each year after the transition age. Estimates are also presented in Table 1 of the cumulative percentage loss from age 35, denoted R_k. R_k will be called the "age factor."

When examining the estimation results it is important to realize that the estimate of the transition age k^* and the estimate of the quadratic parameter δ are collinear. If one is low, the other tends to be low, and vice versa. In other words, sometimes the estimation gives an early transition age and low quadratic curvature, and sometimes it gives a late transition age and high curvature. The best way to see if two estimated equations are similar at the older ages is not to look at the estimates of k^* and δ, but at the implied age factors.

The data that are needed for a specific event in the estimation are observations on the best-ever performance by age beginning with age 35. The track and field data (100, 200, 400, 800, 1,500, 5,000, and 10,000 meters and the high jump) are from *Masters Age Records 2003 Edition*, and the road racing data (5K, 10K, and marathon) are from TACSTATS/USA. Only data for men were used. The data for women were not used because the small sample problem seemed serious at the very old ages for a number of the events. The track and field data give the world record by age for each event. The road racing data, on the other hand, give only the record for a U.S. citizen by age for each event. Ideally, world records should be used instead of just U.S. records, but such data are not available for road racing. Likewise, for swimming the times are for U.S. citizens only, because sufficient data on world records by individual ages are not available for swimming.

The swimming data were obtained from the United States Masters Swimming (USMS) website (*www.usms.org*). Records for long course meters (LCM) and short course yards (SCY) were obtained for both men and women. For LCM there are 17 events, and for SCY there are 18 events. Records were thus obtained for 70 swimming cases. Although data for women were used for swimming, there may be small sample problems at the very old ages.

The chess data are from the World Chess Federation (FIDE). From the FIDE website (*www.fide.com*) it is possible to download chess ratings for about 50,000 players. In most case the player's birth date is also given. The files for October 2003 and April 2004 were downloaded. Women players were excluded, again because of likely small sample problems for women. In addition, a player was excluded if his rating did not change between the two dates. In almost all cases an unchanged rating over a six-month period means the player is not active. The aim was to choose only active players. From the resulting dataset, the best rating was retrieved for each age from age 35 on. One player was excluded, Garry Kasparov. His rating was such an outlier that no sensible line could have been fitted using this value and his age, 41. The chess data are different from the data for the other events in that the observations are not world or U.S. records. The observation for a given age is the best rating for an active player at a particular date, not necessarily the best ever. Small sample problems may thus be more serious for chess than for the other events.

The first phase of the estimation work was to obtain estimates of α, δ, and k^* for each separate event. The second phase was to pool the events whose estimates of α, δ, and k^* were similar.

RESULTS

It will be useful to begin with the pooled estimates, which are presented in Table 1. For the track and road racing data the pooling was for 100, 200, and 400 meters (Sprint) and for all others (Run). For swimming the pooling was for men and women separately and for three distances each (M50, M100, M200+, W50, W100, W200+). Table 1 presents the coefficient estimates and the implied age factors for ages 40, 50, 60, 70, 80, 90, and 100.[3] Also presented are the number of observations and the maximum age used in the estimation. Finally, 10-year rates of decline are presented.[4]

Consider first Sprint versus Run. For Sprint the estimates of α and k^* are 0.0059 and 77.5, and for Run they are 0.0080 and 75.1. The estimate of δ is slightly smaller for Sprint. Decline is thus less for Sprint than for Run: decline is larger for the longer distances. At age 80 the age factor is 1.32 for Sprint and 1.49 for Run. The 10-year rate of decline at age 80 is 7.1 percent for Sprint and 12.8 percent for Run.

The results for men's swimming show the collinearity between the estimates of k^* and δ discussed above. The estimate of k^* is low for M200+ relative to the estimates for M50 and M100, as is the estimate of δ. The age factors are similar for the three categories through age 60, and after that the age factors are generally larger for the longer distances, as is the case for Sprint versus Run discussed above.[5]

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[3] For ease of comparison, the R_k values in Table 1 for the high jump and chess are reciprocals of the actual values.

[4] For example, the 10-year rate of decline for end age 50 is $100(R_{50}/R_{40}-1)$. These rates were computed using unrounded values of R, not the values rounded to two decimal places in Table 1.

[5] The age factors are, however, smaller for M200+ versus M100 for ages 90 and 100, but this may be due to small sample problems at the very old ages.

The results for women swimmers are more problematic because of likely small sample problems at the very old ages. For W100 and W200+ the estimated value for k^* was 35, which means that there is no linear segment before the quadratic. The age-80 results are similar for women and men in that the age factors increase with distance. Also, the age-80 age factors for women are larger than they are for men. For example, for 200+ R_{80} is 1.70 for women and 1.55 for men, a 9.7 percent difference.

The results for the high jump are similar to those for Run regarding the implied age factors.

The results for chess are striking in that they show much smaller rates of decline than for any of the physical activities. For example, the age factor for age 80 for chess is 1.11, which compares to the next smallest age-80 age factor of 1.31 for M50.

Table 2 presents the individual estimates for all the cases. The format is the same as that for Table 1. The first thing to look for are estimates that are out of line with the others, and there are actually very few in Table 2. For the marathon the estimate of k^* is somewhat lower than for the other running events, although the estimate of δ is also lower. This is discussed below. For swimming the largest differences are for the butterfly (FL) for both men and women, where the age factors are generally larger than for the others. The maximum ages for FL are generally lower than for the others, which may reflect a more serious small sample problem for FL than for the others.

Regarding pooling, it seems clear from Table 2 that the 100 meter, 200 meter, and 400 meter track results are close enough to warrant pooling. For the remaining running events the main question is what to do about the marathon. Aside from the results for the marathon, the results for the other running events are fairly close. The marathon is a case of a low estimated value of k^* going along with a low estimated value of δ. Less confidence can be placed on the estimates for the marathon than for the other running events because the maximum age is only 92 for the marathon. It is the case, however, that the values of R_k for the marathon are fairly similar to those for the other running events, and primarily because of this, the marathon was pooled with the other running events.

Regarding swimming, it is generally the case in Table 2 that the age factors increase with distance, especially at the older ages. The age factors are also generally larger for women than for men, again especially at the older ages. The pooling in Table 1 is designed to pick up these differences.

Using the pooled estimates in Table 1, Table 3 presents the age factors R_k for ages 35 through 100. These age factors have already been presented in Table 1 for ages 40, 50, 60, 70, 80, 90, and 100. Although the estimates have been presented through age 100, not much confidence should be placed on the estimates in the 90s because of the small sample problem. The true curvature is not likely to be pinned down very well at the very old ages.

To get a picture of the different rates of decline, Figure 1 shows plots of the age factors from Table 3 for Sprint, Run, M100, and Chess. These plots show clearly the much smaller rates of decline for chess. Sprint and M100 are similar through age 75, at which point the rates of decline for M100 become larger.

DISCUSSION

The 10-year rates of decline in Table 1 provide useful measures to compare to other studies.[6] For chess the 10-year rate of decline at age 80 is about 4 percent. As noted above, this is much smaller than for any of the physical activities. One study of chess (Charness, Krampe, & Mayr, 1996) shows even smaller 10-year rates of decline than those in Table 1 for ages 45-55 and 55-65, but the smaller estimates may be due to cross-section bias since record bests by age were not used. Similarly, a study of the game of GO (Masunaga & Horn, 2001) showed no decline with age, which may also be due to cross-section bias. The estimated nonlinear (quadratic) decline for chess at the older ages in Table 1 is, however, consistent with nonlinear decline after age 65 found in Finkel, Reynolds, McArdle, Gatz, & Pedersen (2003) for cognitive measures with a large speed component.

Regarding physical activities, estimates commonly reported for the decline in $\dot{V}o_{2max}$, the maximum rate of oxygen flow for an individual, are 5 to 10 percent per decade (Heath, Hagberg, Ehsani, & Holloszy, 1981, Rogers, Hagberg, Martin III, Eksani, & Holloszy, 1990, Rosen, Sorkin, Goldberg, Hagberg, & Katzel, 1998, Trappe, Costill, Vukovich, Jones, & Melham, 1996). An exception is Pollock, Foster, Knapp, Rod, & Schmidt (1987), where no decline was found in a 10-year follow-up for a group of highly competitive athletes. Although $\dot{V}o_{2max}$ and running performance are far from perfectly correlated (Noakes, 2003), the correlation is high enough to provide an interesting basis of comparison regarding rates of

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[6] Because of the collinearity between the estimates of k^* and δ mentioned above, the following discussion focuses on the 10-year rates of decline in Table 1 rather than on the individual estimates of k^* and δ.

decline. The 10-year rates of decline in Table 1 for the physical events for men are generally between 5 and 10 percent through age 70, although for M50 the rate is only 4.3 percent. After age 70 the quadratic effects become important, and by age 90 the 10-year rates are between 32.5 and 50.1 percent. For women the 5 to 10 percent range is relevant only through about age 60. The present results thus show that the 5 to 10 percent range is a reasonable approximation through age 70 for men and age 60 for women, but not after that. The advantage of the approach in this study is that rates of decline can be estimated for ages much older than 70, where it seems clear that the decline is more than 5 to 10 percent per decade.

The results for the exponential model in Stones & Kozma (1980) (their Table 2) show a yearly rate of decline of 0.9 percent for 200 meters and 1.2 percent for the marathon, thus showing a faster rate of decline for the longer distance. In Table 1 the yearly rate of decline up to about age 75 is 0.59 percent for Sprint (which includes 200 meters) and 0.80 percent for Run (which includes the marathon). The estimated rates of decline are thus smaller in this study than in Stones & Kozma (1980) for ages below 75, but both studies show a faster rate of decline for the longer distances. In this study, unlike in Stones & Kozma (1980), the rates of decline increase at the older ages (because of the quadratic specification), and so at some point they become larger than those in Stones & Kozma (1980). However, even as they become larger, it is still the case that the rates are larger for the longer distances. The results in Moore (1975) when converted to percents (Baker, Tang, & Turner, 2003, p. 60) are 0.91 percent for 200 meters and 1.11 percent for the marathon, again larger than those in this study except at the older ages but also showing more decline at the longer distance.

The results in Table 1 for swimming for both men and women also generally show larger rates of decline at the longer distances. This is not true for ages above about 90, but the estimates for ages above 90 are less reliable than the others because of the small sample problem. They also generally show larger rates of decline for women than for men, although again not at the very old ages. The larger rates of decline at the longer distances and the larger rates for women versus men are consistent with results reviewed in Tanaka & Seals (2003). The results in Table 1 are probably not precise enough (because of the small sample problem) to form any conclusions about swimming versus running. Comparing the age factors, Sprint is fairly close to M50 for all but the very old ages, as is Run versus M200+. The results are also probably not precise enough to conclude whether women are more affected by increasing distances than are men in their rates of decline.

Regarding future research, as more and more older people compete in the various events, more reliable estimates will be able to be obtained for the older ages. In addition, as more women compete, the estimates for women will become more reliable. It may also be possible to add other events. For example, Crash B rowing is an event that in a few years may have enough data to estimate rates of decline in rowing.

New data for cognitive activities are obviously harder to come by. Chess has the advantage that very good records are kept by age. Many cognitive skills are, of course, involved in playing chess, and so the chess results in this study cannot be taken as measuring rates of decline in any one narrow skill. To use the methodology in this paper to analyze narrower cognitive skills, best scores by age are needed for specific tests that have been taken by many people of many ages. If such data can be found or created, it will be interesting to see if the estimated rates of decline are similar those estimated in this study for chess.

The estimated rates of decline in this study may be useful benchmarks for other studies. As noted in the introduction, they are based on very large samples and on large age ranges. They are also free from traditional forms of selection bias. If in a cross-section study the measured rates of decline are smaller than the present estimated rates for similar activities, this may be cause for concern regarding possible selection bias.

It is, of course, not clear whether the rates of decline in Table 1 are relevant for any specific individual. All but the very elite athletes have lower capacity *levels* than the record levels, but the key question is whether they have similar *rates* of decline as those estimated from the age records. Does a person of average talent who is not sick or injured and who is in good shape slow down at a similar percent rate as elite athletes? The estimates in this study are obviously of more use if the variation in rates of decline across healthy individuals is small than if it is large. The key limitation of any study using best-performance records by age is the need to assume that this variation is small in order to apply the estimated rates of decline to specific individuals.

Finally, another limitation of using best-performance records by age is that the data do not reveal the causes or mechanisms of the age trends. Rates of decline can be compared across events, as done in this study, but there is no information in the data regarding causes.

APPENDIX

The postulated formula for b_k, the log of the biological minimum time for age k for a particular event, is

$$b_k = \begin{cases} \beta + \alpha k, & 35 \leq k \leq k^* \\ \gamma + \theta k + \delta k^2, & k > k^* \end{cases} \tag{2}$$

with the restrictions

$$\begin{aligned} \gamma &= \beta + \delta k^{*2} \\ \theta &= \alpha - 2\delta k^* \end{aligned} \tag{3}$$

The two restrictions force the linear and quadratic segments to touch and to have the same first derivative at k^*. The unrestricted parameters to estimate are the intercept, β, the slope of the linear segment, α, the age at which the line changes from linear to quadratic, k^*, and the quadratic parameter, δ.

The equation that is estimated for a given event, where r_k is the log of the observed record time for age k, is

$$r_k = \beta + \alpha k + \delta d_k (k^{*2} - 2k^* k + k^2) + \epsilon_k, \tag{4}$$

where $d_k = 0$ if $k \leq k^*$ and $d_k = 1$ if $k > k^*$. k ranges over the non-dominated observations. Since, as discussed in the text, ϵ_k can never be negative, an estimation method is needed that insures that the estimated value of ϵ_k, denoted $\hat{\epsilon}_k$, will never to be negative. This was done by choosing the estimates of the parameter values in equation 4 to minimize the sum of squared residuals subject to the restrictions that all the estimated errors are non-negative. In addition, the estimated error for the first observation is forced to be zero, under the assumption that the measurement error for the first observations is zero.

Equation 4 is nonlinear in the parameters β, α, k^*, and δ. These parameters were estimated using a nonlinear optimization algorithm by minimizing the weighted sum $\sum_k \lambda_k \hat{\epsilon}_k^2$, where λ_k is equal to 1 if $\hat{\epsilon}_k \geq 0$ and is equal to a number greater than 1 if $\hat{\epsilon}_k < 0$, where $\hat{\epsilon}_k$ is the estimated error for observation k. This penalizes negative errors more than non-negative ones. In the estimation work a value of 500 was used for λ_k when $\hat{\epsilon}_k$ was less than zero. This was large enough to make nearly all the estimated errors non-negative at the optimum. To insure that the estimated error for the first observations is close to zero, a value of 500 was used for λ_k when k is the first observation.

The estimates for a number of the cases are sensitive to whether or not the first observation is forced to be on the line (i.e., whether or not the estimated error for the first observation is forced to be zero). If this restriction is not imposed, some of the lines imply times that are unrealistically low for ages near 35 (e.g., times that are considerably below the current overall world record). If the measurement error is small for the first observation used, then the current procedure is justified.

The restrictions in equation 3 that are imposed in the estimation are examples of polynomial spline restrictions (Poirier, 1976). The restriction that all the estimated errors be non-negative is common in the estimation of frontier production functions (Aigner & Chu, 1968, Schmidt, 1976). The added complication here is that equation 4 is nonlinear in parameters. For linear equations the estimation problem can be set up as a quadratic programming problem and solved by standard methods, but for nonlinear equations a procedure like the one described above must be used. There is no obvious way to test the hypothesis that the coefficients for one event equal those for another. The assumption of independent and identically distributed errors is not appropriate in this context. In practice, the estimated errors are much larger on average at the old ages, even after excluding the dominated times, which reflects the small sample problem. Comparisons have to be made by looking for patterns across the various cases rather than by formal hypothesis testing.

The values of the age factors, R_k, are computed as follows. Let $\hat{r}_k$ denote the predicted value of r_k from equation 4 using the estimated values of β, α, k^*, and δ and zero values for the error term for $k = 35, \ldots, 100$. Then R_k is

$$R_k = e^{\hat{r}_k}/e^{\hat{r}_{35}}, \quad k = 35, \ldots, 100 \ . \tag{5}$$

It should finally be noted that when pooling is done, a different estimate of β in equation 4 is obtained for each event, but the estimates of α, δ, and k^* are constrained to be the same across events. When using the nonlinear optimization algorithm for pooling, the estimated error for the first observation for each of the separate events was forced to be zero and all the estimated errors were forced to be non-negative (or nearly so).

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Table 1 *Coefficient estimates and implied age factors for 10 cases*

Event	**Estimates**			**Age Factors**							No. Obs.	Max Age
	$\hat{\alpha}$	$\hat{k}^*$	$\hat{\delta}$	R_{40}	R_{50}	R_{60}	R_{70}	R_{80}	R_{90}	R_{100}		
Sprint	0.0059	77.5	0.00158	1.03	1.09	1.16	1.23	1.32	1.77	3.25	119	98
Run	0.0080	75.1	0.00164	1.04	1.13	1.22	1.32	1.49	2.24	4.68	267	96
M50	0.0042	70.8	0.00089	1.02	1.07	1.11	1.16	1.31	1.76	2.83	256	100
M100	0.0050	69.8	0.00113	1.03	1.08	1.13	1.19	1.41	2.09	3.88	319	100
M200+	0.0036	53.4	0.00039	1.02	1.06	1.11	1.26	1.55	2.05	2.94	574	100
W50	0.0050	58.4	0.00047	1.03	1.08	1.13	1.27	1.56	2.11	3.13	231	92
W100	-0.0019	35.0	0.00029	1.00	1.04	1.15	1.34	1.66	2.19	3.05	263	94
W200+	0.0018	35.0	0.00022	1.01	1.08	1.20	1.40	1.70	2.16	2.88	542	94
HJ	-0.0088	70.6	-0.00075	1.04	1.14	1.24	1.36	1.58	2.14	3.37	34	96
Chess	-0.0019	72.3	-0.00032	1.01	1.03	1.05	1.07	1.11	1.22	1.44	10	94

	10-year Rates of Decline					
	End Age					
	50	60	70	80	90	100
Sprint	6.1	6.1	6.1	7.1	34.3	84.2
Run	8.4	8.4	8.4	12.8	50.1	108.5
M50	4.3	4.3	4.3	12.6	34.5	60.8
M100	5.1	5.1	5.1	18.3	48.3	86.0
M200+	3.7	5.5	13.5	22.6	32.5	43.2
W50	5.1	5.2	11.9	23.0	35.2	48.5
W100	4.0	10.3	17.0	24.0	31.5	39.5
W200+	6.4	11.3	16.4	21.7	27.2	33.0
HJ	9.2	9.2	9.2	16.6	35.4	57.2
Chess	1.9	1.9	1.9	3.8	10.4	17.6

Notes:
- Sprint = 100, 200, and 400 meter track.
- Run = all running except 100, 200, and 400 meter track.
- M50 = 50 meter and yard swimming events, men.
- M100 = 100 meter and yard swimming events, men.
- M200+ = all other swimming events, men.
- W50 = 50 meter and yard swimming events, women.
- W100 = 100 meter and yard swimming events, women.
- W200+ = all other swimming events, women.
- HJ = high jump.

Table 2 *Coefficient estimates and implied age factors for each individual event*

Event	**Estimates**			**Age Factors**							No.	Max
	$\hat{\alpha}$	$\hat{k}^*$	$\hat{\delta}$	R_{40}	R_{50}	R_{60}	R_{70}	R_{80}	R_{90}	R_{100}	Obs.	Age
Sprint												
100M	0.0063	79.3	0.00179	1.03	1.10	1.17	1.25	1.33	1.73	3.24	36	98
200M	0.0071	76.5	0.00163	1.04	1.11	1.19	1.28	1.41	1.99	3.91	40	98
400M	0.0057	73.5	0.00168	1.03	1.09	1.15	1.22	1.38	2.15	4.68	43	98
Run												
800M	0.0085	73.3	0.00144	1.04	1.14	1.24	1.35	1.56	2.39	4.85	42	95
1500M	0.0088	77.1	0.00241	1.05	1.14	1.25	1.36	1.52	2.43	6.29	46	96
5000M	0.0079	71.8	0.00117	1.04	1.13	1.22	1.32	1.55	2.28	4.26	40	95
10000M	0.0087	78.6	0.00245	1.04	1.14	1.24	1.36	1.49	2.22	5.43	38	94
5K	0.0073	70.9	0.00146	1.04	1.12	1.20	1.29	1.57	2.54	5.52	34	95
10K	0.0064	72.5	0.00184	1.03	1.10	1.17	1.25	1.48	2.50	6.10	42	94
MA	0.0084	63.6	0.00062	1.04	1.13	1.23	1.37	1.72	2.44	3.90	25	92

Notes:

- M = meters, K = kilometers.
- M events are track, K events are road racing.

Table 2 (continued)

Event	**Estimates**			**Age Factors**							No.	Max
	$\hat{\alpha}$	$\hat{k}^*$	$\hat{\delta}$	R_{40}	R_{50}	R_{60}	R_{70}	R_{80}	R_{90}	R_{100}	Obs.	Age
Swimming, LCM, Men												
50FR	0.0021	76.6	0.00213	1.01	1.03	1.05	1.08	1.13	1.65	3.69	26	100
100FR	0.0048	73.7	0.00163	1.02	1.08	1.13	1.18	1.33	2.01	4.24	34	100
200FR	0.0054	68.8	0.00096	1.03	1.08	1.14	1.21	1.44	2.07	3.61	32	96
400FR	0.0033	59.1	0.00054	1.02	1.05	1.09	1.20	1.47	2.01	3.08	27	96
800FR	0.0031	55.1	0.00045	1.02	1.05	1.09	1.23	1.51	2.04	3.01	26	96
1500FR	0.0021	52.1	0.00045	1.01	1.03	1.08	1.24	1.56	2.13	3.19	25	96
50BA	0.0068	72.5	0.00118	1.03	1.11	1.18	1.27	1.45	2.09	3.80	33	100
100BA	0.0076	73.4	0.00124	1.04	1.12	1.21	1.31	1.49	2.14	3.94	35	100
200BA	0.0086	68.5	0.00080	1.04	1.14	1.24	1.35	1.64	2.33	3.89	31	100
50BR	0.0061	71.3	0.00129	1.03	1.10	1.17	1.24	1.45	2.20	4.31	33	96
100BR	0.0072	73.4	0.00170	1.04	1.11	1.20	1.29	1.49	2.38	5.35	35	96
200BR	0.0080	74.3	0.00162	1.04	1.13	1.22	1.33	1.51	2.31	4.89	36	95
50FL	0.0043	64.3	0.00098	1.02	1.07	1.11	1.20	1.54	2.41	4.59	24	91
100FL	0.0107	58.1	0.00050	1.05	1.17	1.31	1.56	2.05	2.98	4.78	25	91
200FL	0.0051	46.6	0.00039	1.03	1.08	1.22	1.48	1.95	2.78	4.27	24	91
200IM	0.0099	72.5	0.00127	1.05	1.16	1.28	1.41	1.67	2.53	4.94	29	91
400IM	0.0068	54.0	0.00039	1.03	1.11	1.20	1.40	1.76	2.40	3.52	28	91
Swimming, SCY, Men												
50FR	0.0030	73.6	0.00160	1.02	1.05	1.08	1.11	1.22	1.81	3.70	35	100
100FR	0.0030	66.9	0.00104	1.01	1.05	1.08	1.12	1.37	2.05	3.79	40	100
200FR	0.0049	67.9	0.00098	1.02	1.08	1.13	1.19	1.44	2.11	3.77	37	100
500FR	0.0033	52.4	0.00038	1.02	1.05	1.11	1.26	1.55	2.05	2.92	35	95
1000FR	0.0032	59.9	0.00071	1.02	1.05	1.08	1.20	1.54	2.27	3.87	33	96
1650FR	0.0043	55.7	0.00050	1.02	1.07	1.12	1.28	1.62	2.26	3.49	29	93
50BA	0.0042	56.7	0.00046	1.02	1.06	1.11	1.25	1.55	2.09	3.10	35	95
100BA	0.0079	70.5	0.00109	1.04	1.13	1.22	1.32	1.58	2.34	4.31	42	98
200BA	0.0079	72.6	0.00155	1.04	1.13	1.22	1.32	1.56	2.47	5.34	38	94
50BR	0.0069	72.4	0.00133	1.04	1.11	1.19	1.28	1.48	2.22	4.34	37	96
100BR	0.0072	70.6	0.00132	1.04	1.11	1.20	1.29	1.55	2.44	4.97	38	96
200BR	0.0085	72.2	0.00157	1.04	1.14	1.24	1.35	1.61	2.62	5.84	43	94
50FL	0.0045	68.9	0.00174	1.02	1.07	1.12	1.17	1.52	2.79	7.27	33	91
100FL	0.0033	55.7	0.00077	1.02	1.05	1.10	1.31	1.83	2.97	5.63	33	90
200FL	0.0056	56.6	0.00080	1.03	1.09	1.16	1.40	1.99	3.33	6.52	30	90
100IM	0.0069	65.8	0.00088	1.04	1.11	1.19	1.30	1.63	2.45	4.39	37	94
200IM	0.0095	74.9	0.00246	1.05	1.15	1.27	1.39	1.63	2.95	8.74	34	91
400IM	0.0086	67.1	0.00116	1.04	1.14	1.24	1.36	1.79	2.95	6.13	37	90

Notes:

- LCM = long course meters, SCY = short course yards.
- FR = free, BA = back, BR = breast, FL = fly, IM = individual medley.

Table 2 (continued)

Event	**Estimates**			**Age Factors**							No.	Max
	$\hat{\alpha}$	$\hat{k}^*$	$\hat{\delta}$	R_{40}	R_{50}	R_{60}	R_{70}	R_{80}	R_{90}	R_{100}	Obs.	Age
Swimming, LCM, Women												
50FR	0.0013	43.9	0.00030	1.01	1.03	1.12	1.29	1.57	2.05	2.83	35	92
100FR	0.0044	47.3	0.00030	1.02	1.07	1.17	1.36	1.68	2.19	3.05	29	94
200FR	0.0038	42.5	0.00025	1.02	1.07	1.19	1.38	1.69	2.17	2.93	33	94
400FR	0.0071	69.5	0.00125	1.04	1.11	1.20	1.28	1.58	2.50	5.08	32	92
800FR	0.0015	43.3	0.00031	1.01	1.04	1.13	1.31	1.62	2.13	2.98	31	92
1500FR	0.0116	76.9	0.00206	1.06	1.19	1.34	1.50	1.72	2.69	6.36	21	91
50BA	0.0035	50.6	0.00026	1.02	1.05	1.12	1.25	1.47	1.82	2.38	23	91
100BA	0.0073	62.0	0.00050	1.04	1.11	1.20	1.33	1.63	2.20	3.28	27	90
200BA	0.0062	35.7	0.00013	1.03	1.13	1.26	1.45	1.71	2.07	2.57	34	91
50BR	0.0085	66.7	0.00092	1.04	1.14	1.24	1.36	1.73	2.64	4.85	20	90
100BR	0.0097	61.1	0.00047	1.05	1.16	1.27	1.46	1.83	2.52	3.83	29	91
200BR	0.0105	64.5	0.00057	1.05	1.17	1.30	1.47	1.83	2.57	4.02	34	90
50FL	0.0051	55.2	0.00076	1.03	1.08	1.16	1.41	2.01	3.33	6.40	26	88
100FL	0.0095	48.3	0.00038	1.05	1.15	1.33	1.67	2.24	3.26	5.11	25	89
200FL	0.0049	46.5	0.00045	1.02	1.08	1.23	1.52	2.07	3.08	5.01	29	86
200IM	0.0038	46.7	0.00040	1.02	1.06	1.18	1.42	1.86	2.63	4.04	33	91
400IM	0.0081	60.8	0.00067	1.04	1.13	1.22	1.40	1.84	2.75	4.69	29	89
Swimming, SCY, Women												
50FR	0.0068	60.5	0.00046	1.03	1.11	1.18	1.32	1.62	2.17	3.20	33	91
100FR	0.0048	53.4	0.00044	1.02	1.07	1.15	1.33	1.69	2.34	3.52	33	91
200FR	0.0082	48.2	0.00022	1.04	1.13	1.26	1.48	1.80	2.29	3.05	32	92
500FR	0.0101	64.9	0.00052	1.05	1.16	1.29	1.45	1.78	2.43	3.69	32	92
1000FR	0.0089	60.6	0.00042	1.05	1.14	1.25	1.42	1.75	2.35	3.44	26	92
1650FR	0.0096	63.2	0.00044	1.05	1.15	1.27	1.43	1.74	2.33	3.40	25	90
50BA	0.0061	59.0	0.00050	1.03	1.10	1.16	1.31	1.63	2.25	3.42	29	91
100BA	0.0113	67.0	0.00058	1.06	1.18	1.33	1.49	1.83	2.52	3.90	33	91
200BA	0.0065	36.6	0.00016	1.04	1.13	1.28	1.50	1.80	2.24	2.87	28	90
50BR	0.0093	65.8	0.00083	1.05	1.15	1.26	1.40	1.79	2.71	4.82	28	90
100BR	0.0101	59.1	0.00044	1.05	1.16	1.29	1.50	1.91	2.65	4.02	30	88
200BR	0.0104	62.9	0.00059	1.05	1.17	1.30	1.48	1.90	2.74	4.44	33	90
50FL	0.0057	56.9	0.00076	1.03	1.09	1.16	1.39	1.94	3.14	5.93	37	91
100FL	0.0070	45.7	0.00038	1.04	1.12	1.29	1.60	2.15	3.11	4.87	27	90
200FL	0.0189	70.1	0.00074	1.10	1.33	1.60	1.94	2.51	3.79	6.60	24	90
100IM	0.0117	68.9	0.00098	1.06	1.19	1.34	1.51	1.91	2.94	5.49	30	92
200IM	0.0126	66.0	0.00071	1.07	1.21	1.37	1.57	2.03	3.01	5.15	32	92
400IM	0.0051	49.9	0.00045	1.03	1.08	1.19	1.43	1.89	2.73	4.32	34	90

Notes:

- LCM = long course meters, SCY = short course yards.
- FR = free, BA = back, BR = breast, FL = fly, IM = individual medley.

Table 3 *Implied age factors (R_k) using coefficient estimates in Table 1*

Age	Sprint	Run	M50	M100	M200+	W50	W100	W200+	HJ	Chess
35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
36	1.006	1.008	1.004	1.005	1.004	1.005	0.998	1.002	1.009	1.002
37	1.012	1.016	1.009	1.010	1.007	1.010	0.997	1.004	1.018	1.004
38	1.018	1.024	1.013	1.015	1.011	1.015	0.997	1.007	1.027	1.006
39	1.024	1.033	1.017	1.020	1.015	1.020	0.997	1.011	1.036	1.008
40	1.030	1.041	1.021	1.025	1.018	1.025	0.998	1.015	1.045	1.009
41	1.036	1.049	1.026	1.030	1.022	1.030	0.999	1.019	1.054	1.011
42	1.042	1.058	1.030	1.035	1.026	1.035	1.001	1.024	1.063	1.013
43	1.048	1.066	1.035	1.041	1.030	1.041	1.003	1.029	1.073	1.015
44	1.054	1.075	1.039	1.046	1.033	1.046	1.007	1.035	1.082	1.017
45	1.061	1.084	1.043	1.051	1.037	1.051	1.010	1.041	1.092	1.019
46	1.067	1.092	1.048	1.056	1.041	1.056	1.015	1.048	1.101	1.021
47	1.073	1.101	1.052	1.061	1.045	1.061	1.019	1.055	1.111	1.023
48	1.079	1.110	1.057	1.067	1.048	1.067	1.025	1.063	1.121	1.025
49	1.086	1.119	1.061	1.072	1.052	1.072	1.031	1.071	1.130	1.027
50	1.092	1.128	1.066	1.077	1.056	1.077	1.038	1.080	1.140	1.029
51	1.099	1.137	1.070	1.083	1.060	1.083	1.046	1.089	1.150	1.031
52	1.105	1.146	1.075	1.088	1.064	1.088	1.054	1.099	1.161	1.032
53	1.112	1.155	1.079	1.094	1.068	1.094	1.063	1.110	1.171	1.034
54	1.118	1.165	1.084	1.099	1.072	1.099	1.072	1.121	1.181	1.036
55	1.125	1.174	1.089	1.104	1.076	1.105	1.082	1.133	1.191	1.038
56	1.131	1.184	1.093	1.110	1.082	1.110	1.093	1.145	1.202	1.040
57	1.138	1.193	1.098	1.115	1.089	1.116	1.105	1.158	1.212	1.042
58	1.145	1.203	1.102	1.121	1.096	1.121	1.118	1.172	1.223	1.044
59	1.152	1.212	1.107	1.127	1.104	1.127	1.131	1.187	1.234	1.046
60	1.158	1.222	1.112	1.132	1.114	1.134	1.145	1.202	1.245	1.048
61	1.165	1.232	1.117	1.138	1.124	1.142	1.160	1.218	1.256	1.050
62	1.172	1.242	1.121	1.144	1.135	1.151	1.176	1.234	1.267	1.052
63	1.179	1.252	1.126	1.149	1.147	1.161	1.193	1.252	1.278	1.054
64	1.186	1.262	1.131	1.155	1.161	1.172	1.211	1.270	1.289	1.056
65	1.193	1.272	1.136	1.161	1.175	1.185	1.230	1.289	1.300	1.058
66	1.200	1.283	1.141	1.166	1.190	1.199	1.249	1.309	1.312	1.060
67	1.207	1.293	1.145	1.172	1.207	1.214	1.270	1.330	1.323	1.062
68	1.214	1.303	1.150	1.178	1.224	1.231	1.292	1.352	1.335	1.064
69	1.221	1.314	1.155	1.184	1.243	1.249	1.315	1.375	1.347	1.066
70	1.229	1.324	1.160	1.190	1.263	1.269	1.340	1.399	1.359	1.068
71	1.236	1.335	1.165	1.198	1.285	1.290	1.365	1.423	1.371	1.070
72	1.243	1.346	1.172	1.208	1.308	1.312	1.392	1.449	1.385	1.072
73	1.250	1.357	1.180	1.222	1.332	1.336	1.420	1.476	1.401	1.074
74	1.258	1.368	1.191	1.238	1.358	1.362	1.450	1.505	1.419	1.077
75	1.265	1.379	1.204	1.258	1.386	1.390	1.481	1.534	1.440	1.081

Table 3 (continued)

Age	Sprint	Run	M50	M100	M200+	W50	W100	W200+	HJ	Chess
76	1.273	1.392	1.219	1.281	1.415	1.420	1.514	1.565	1.463	1.085
77	1.280	1.410	1.237	1.307	1.445	1.452	1.548	1.597	1.489	1.090
78	1.288	1.432	1.257	1.336	1.478	1.485	1.584	1.630	1.518	1.095
79	1.300	1.460	1.280	1.370	1.513	1.521	1.622	1.665	1.549	1.102
80	1.315	1.494	1.306	1.407	1.549	1.560	1.662	1.702	1.584	1.109
81	1.336	1.533	1.335	1.449	1.588	1.601	1.703	1.740	1.622	1.117
82	1.360	1.578	1.366	1.495	1.629	1.644	1.747	1.779	1.663	1.126
83	1.390	1.630	1.401	1.547	1.672	1.691	1.793	1.821	1.707	1.135
84	1.425	1.689	1.439	1.603	1.718	1.740	1.841	1.864	1.756	1.145
85	1.465	1.757	1.481	1.666	1.766	1.792	1.892	1.909	1.809	1.156
86	1.511	1.832	1.527	1.735	1.817	1.848	1.945	1.956	1.866	1.168
87	1.564	1.918	1.577	1.811	1.871	1.907	2.001	2.005	1.927	1.181
88	1.623	2.014	1.632	1.894	1.928	1.970	2.059	2.056	1.994	1.195
89	1.691	2.122	1.692	1.986	1.988	2.037	2.121	2.109	2.066	1.209
90	1.766	2.243	1.757	2.087	2.052	2.108	2.186	2.165	2.144	1.225
91	1.851	2.378	1.827	2.198	2.120	2.184	2.254	2.223	2.228	1.241
92	1.946	2.530	1.904	2.320	2.191	2.265	2.325	2.284	2.319	1.259
93	2.052	2.701	1.988	2.455	2.267	2.351	2.400	2.347	2.418	1.277
94	2.171	2.893	2.079	2.603	2.347	2.442	2.479	2.414	2.524	1.297
95	2.304	3.108	2.179	2.767	2.432	2.539	2.563	2.483	2.639	1.318
96	2.453	3.350	2.287	2.947	2.522	2.643	2.650	2.556	2.763	1.340
97	2.620	3.624	2.404	3.147	2.617	2.754	2.742	2.631	2.897	1.363
98	2.807	3.932	2.533	3.367	2.718	2.872	2.839	2.710	3.043	1.388
99	3.017	4.281	2.672	3.611	2.825	2.998	2.941	2.793	3.200	1.413
100	3.254	4.676	2.825	3.882	2.938	3.132	3.049	2.880	3.371	1.441

- See notes to Table 1.

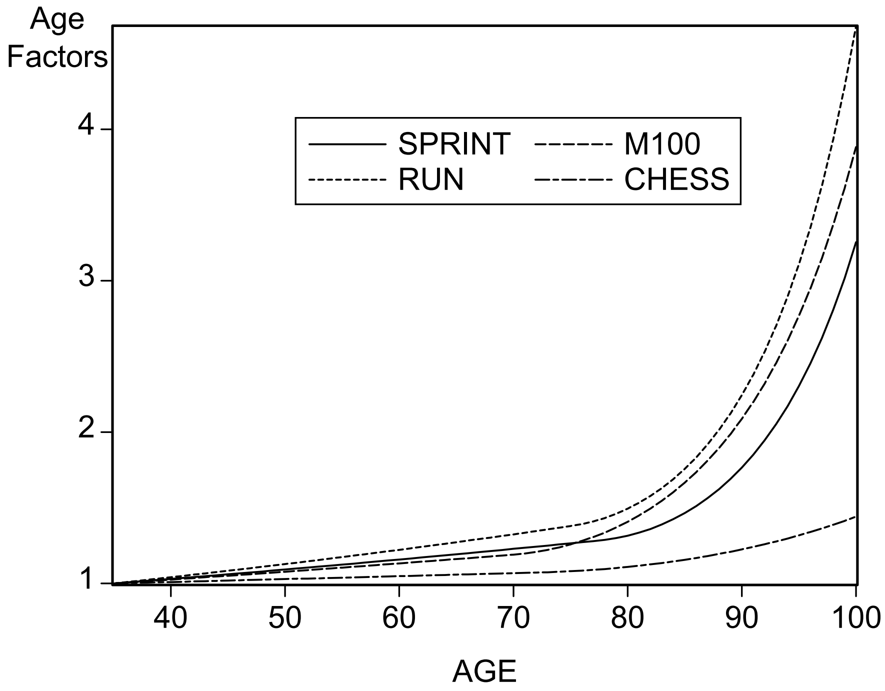

Figure 1 *Estimated age factors for four cases and ages 35-100. Age 35 = 1.0. Data are in Table 3.*